2) $\forall n \in \mathbb{N}$; $v_n = \frac{1}{u_n^2}$

$\Rightarrow u_n^2 = \frac{1}{v_n}$

$\Rightarrow u_n = \frac{1}{\sqrt{v_n}}$ car $0 < u_n < 1$

$\Rightarrow \boxed{u_n = \frac{1}{\sqrt{n+1}} \quad \forall n \in \mathbb{N}}$

3) $\forall n \in \mathbb{N}$; $S_n = \sum_{k=0}^{n} u_k$

a) $\boxed{S_0 = u_0 = 1}$

$\boxed{S_1 = u_0 + u_1 = 1 + \frac{1}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}}$

$\boxed{S_2 = u_0 + u_1 + u_2 = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}}$

b) $\forall k \in \mathbb{N}$; $\sqrt{k+2} - \sqrt{k+1} \times \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}}$

$= \frac{k+2-(k+1)}{\sqrt{k+2} + \sqrt{k+1}}$

$= \frac{1}{\sqrt{k+2} + \sqrt{k+1}}$

Qarray.tn -Agra Blech- Qarray.tn